Pythagoras Theorem
In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

Given
ΔACB is a right angled triangle in which \( \angle C = 90^\circ \) and \( m\angle B = a \), \( m\angle A = b \) and \( m\angle A = c \).

To Prove
\( c^2 = a^2 + b^2 \)

Construction
Draw \( CD \) perpendicular from \( C \) on \( AB \).
Let \( m\angle C = h \), \( m\angle A = x \) and \( m\angle D = y \).
Line segment \( CD \) splits \( \triangle ABC \) into two \( \triangle s \) \( \triangle ADC \) and \( \triangle BDC \) which are separately shown in the figures (ii)-a and (ii)-b respectively.

Proof (Using similar \( \triangle s \))

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>( \triangle ADC ) ( \longrightarrow ) ( \triangle ACB )</td>
<td>Refer to figure(ii)-a and (i)</td>
</tr>
<tr>
<td>( \angle A \equiv \angle A )</td>
<td>Common – self congruent</td>
</tr>
<tr>
<td>( \angle ADC \equiv \angle ACB )</td>
<td>Construction – given, each angle = ( 90^\circ )</td>
</tr>
<tr>
<td>( \angle C \equiv \angle B )</td>
<td>( \angle C ) and ( \angle B ), complements of ( \angle A ).</td>
</tr>
<tr>
<td>( \therefore \triangle ADC \sim \triangle ACB )</td>
<td>Congruency of three angles</td>
</tr>
<tr>
<td>( \therefore \frac{x}{a} = \frac{b}{c} )</td>
<td>(Measures of corresponding sides of similar triangles are proportional)</td>
</tr>
<tr>
<td>( \text{or } x = \frac{b^2}{c} )</td>
<td></td>
</tr>
</tbody>
</table>
Again in \( \triangle BDC \leftrightarrow \triangle BCA \)
1. \( \angle B \equiv \angle B \)
2. \( \angle BDC \equiv \angle BCA \)
3. \( \angle C \equiv \angle A \)

\( \therefore \triangle BDC \sim \triangle BCA \)

From
\[ y = \frac{a}{c} \]
or
\[ y = \frac{a^2}{c} \] \( \cdots \) (ii)

But \( y^2 + x^2 = c^2 \)
\[ \therefore \frac{a^2}{c} + \frac{b^2}{c} = \frac{c^2}{c} \] \( \therefore \)
or \( a^2 + b^2 = c^2 \)

i.e., \( c^2 = a^2 + b^2 \)

**Corollary**

In a right angled \( \triangle ABC \), right angled at \( A \).

(i) \( AB^2 = BC^2 - CA^2 \)

(ii) \( AC^2 = BC^2 - AB^2 \)

**Converse of Pythagoras’ Theorem**

If the square of one side of a triangle is equal to the sum of the squares of the other two sides then the triangle is a right angled triangle.

**Proof**

<table>
<thead>
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<tr>
<td>( \Delta DCB ) is a right-angled triangle.</td>
<td>Construction</td>
</tr>
<tr>
<td>( \therefore (mBD)^2 = a^2 + b^2 )</td>
<td>Pythagoras theorem</td>
</tr>
<tr>
<td>But ( a^2 + b^2 = c^2 )</td>
<td>Given</td>
</tr>
<tr>
<td>( \therefore (mBD)^2 = c^2 )</td>
<td>Taking square root of both sides.</td>
</tr>
<tr>
<td>or ( mBD = c )</td>
<td></td>
</tr>
<tr>
<td>Now in ( \Delta DCB \leftrightarrow \Delta ACB )</td>
<td>Construction</td>
</tr>
<tr>
<td>( CD \equiv CA )</td>
<td></td>
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</tbody>
</table>

Refer to figure (ii)-b and (i)
Common—self congruent
Construction—given, each angle = 90°
\( \angle C \) and \( \angle A \), complements of \( \angle B \)
Congruency of three angles.
(Corresponding sides of similar triangles are proportional).

**Supposition.**

By (i) and (ii)
Multiplying both sides by \( c \).

**Given.** In a \( \triangle ABC \), \( m\overline{AB} = c, m\overline{BC} = a \) and \( m\overline{AC} = b \) such that \( a^2 + b^2 = c^2 \).

**To Prove.** \( \triangle ACB \) is a right angled triangle.

**Construction.** Draw \( \overline{CD} \) perpendicular to \( BC \) such that \( CD \equiv CA \). Join the points \( B \) and \( D \).
\[ \frac{BC}{AB} = \frac{BC}{AB} \]
\[ \therefore \triangle DCB \cong \triangle ACB \]
\[ \therefore \angle DCB = \angle ACB \]
But \( m\angle DCB = 90^\circ \)
\[ \therefore m\angle ACB = 90^\circ \]
Hence the \( \triangle ACB \) is a right-angled triangle.

**Corollary:** Let \( c \) be the longest of the sides \( a, b \) and \( c \) of a triangle.
- If \( a^2 + b^2 = c^2 \), then the triangle is right.

<table>
<thead>
<tr>
<th>Common</th>
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<tbody>
<tr>
<td>Each side = c.</td>
</tr>
<tr>
<td>S.S.S. ( \cong ) S.S.S.</td>
</tr>
<tr>
<td>(Corresponding angles of congruent triangles)</td>
</tr>
<tr>
<td><strong>Construction</strong></td>
</tr>
</tbody>
</table>

- If \( a^2 + b^2 > c^2 \), then the triangle is acute.
- If \( a^2 + b^2 < c^2 \), then the triangle is obtuse.

**Exercise Set 1F**

1. Verify that the \( \triangle \)s having the following measures of sides are right-angled.
   (i) \( a = 5 \text{ cm}, \ b = 12 \text{ cm}, \ c = 13 \text{ cm} \)
   Ans. \((Hyp)^2 = (Perp.)^2 + (Base)^2 \\
   (13)^2 = (12)^2 + (5)^2 \\
   169 = 144 + 25 \\
   169 = 169 \\
   .\therefore \text{The triangle is right-angled.}

   (ii) \( a = 1.5 \text{ cm}, b = 2 \text{ cm}, c = 2.5 \text{ cm} \)
   Ans. \((Hyp)^2 = (Perp.)^2 + (Base)^2 \\
   (2.5)^2 = (1.5)^2 + (2)^2 \\
   6.25 = 2.25 + 4 \\
   6.25 = 6.25 \\
   .\therefore \text{The triangle is right-angled.}

   (iii) \( a = 9 \text{ cm}, b = 12 \text{ cm}, c = 15 \text{ cm} \)
   Ans. \((Hyp)^2 = (Perp.)^2 + (Base)^2 \\
   (15)^2 = (12)^2 + (9)^2 \\
   225 = 144 + 81 \\
   225 = 225 \\
   .\therefore \text{The triangle is right-angled.}

   (iv) \( a = 16 \text{ cm}, b = 30 \text{ cm}, c = 34 \text{ cm} \)
   Ans. \((Hyp)^2 = (Perp.)^2 + (Base)^2 \\
   (34)^2 = (30)^2 + (16)^2 \\
   1156 = 900 + 256 \\
   1156 = 1156 \\
   .\therefore \text{The triangle is right-angled.}

2. Verify that \( a^2 + b^2, a^2 - b^2 \) and \( 2ab \) are the measures of the sides of a right angled triangle where \( a \) and \( b \) are any two real numbers \((a > b)\).
   Ans. In right angle triangle.
   \[ H^2 = p^2 + b^2 \]
   \[ (a^2 + b^2)^2 = a^4 + b^2 + 2a^2b^2 \] ............(i)
   \[ (a^2 - b^2)^2 = a^4 + b^4 - 2a^2b^2 \] ............(ii)
   \[ (2ab)^2 = 4a^2b^2 \] .................... (iii)
   Adding (ii) and (iii) we get
   \[ (a^2 - b^2)^2 + (2ab)^2 = a^4 + b^4 - 2a^2b^2 + 4a^2b^2 \]
   \[ = a^4 + b^4 + 2a^2b^2 \] ...............(iv)
   Comparing (i) and (iv), we get
   \[ (a^2 - b^2)^2 + (2ab)^2 = (a^2+b^2)^2 \]
   Hence \( a^2 + b^2, \ a^2 - b^2 \) and \( 2ab \) are measures of the sides of a right angled triangle where \( a^2 + b^2 \) is hypotenuse.

3. The three sides of a triangle are of measure 8, \( x \) and 17 respectively. For what value of \( x \) will it become base of a right angled triangle?
   Ans.
Consider a right angled triangle

With \( \overline{AB} = x \)
\( \overline{BC} = 8 \)

and \( \overline{AC} = 17 \)

If \( x \) is the base of right angled \( \triangle ABC \) then
we know by Pythagoras theorem that
\[
(hyp)^2 = (Base)^2 + (perp)^2
\]
\[
(17)^2 = x^2 + (8)^2
\]
\[
289 = x^2 + 64
\]
\[
x^2 + 64 = 289
\]
\[
x^2 = 289 - 64
\]
\[
x^2 = 225
\]
\[
x = \sqrt{225}
\]
As \( x \) is measure of side
So \( x = 15 \) units

**Proof**

<table>
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<tbody>
<tr>
<td>In right angled triangle</td>
<td>( \overline{CD} = 14 \text{ cm} )</td>
</tr>
<tr>
<td>( \overline{mAC} = 50 \text{ cm} )</td>
<td>( (mAD)^2 = (mAC)^2 - (mCD)^2 )</td>
</tr>
<tr>
<td>( (mAD)^2 = (50)^2 - (14)^2 )</td>
<td>( = 2500 - 196 )</td>
</tr>
<tr>
<td>( = 2304 )</td>
<td>( = \sqrt{2304} )</td>
</tr>
<tr>
<td>( mAD = 18 \text{ cm} )</td>
<td>( \overline{CD} = \frac{1}{2} \overline{mBC} )</td>
</tr>
</tbody>
</table>

\( (mAC)^2 = (mAD)^2 - (mCD)^2 \) (by Pythagoras theorem)

Taking square root of both sides

\[ \text{(ii) Area of } \triangle ABC = \frac{\text{Base } \times \text{ Altitude}}{2} \]
\[ = \frac{28 \times 48}{2} \]
\[ = 14 \times 28 \]
\[ = 672 \text{ sq.cm} \]

**4.** In an isosceles \( \triangle \), the base \( \overline{BC} = 28 \text{ cm} \), and \( \overline{AB} = \overline{AC} = 50 \text{ cm} \).

If \( \overline{AD} \perp \overline{BC} \), then find:

(i) Length of \( \overline{AD} \)

(ii) Area of \( \triangle ABC \)

**Given**
\( mAC = mAB = 50 \text{ cm} \)
\( mBC = 28 \text{ cm} \)
\( \overline{AD} \perp \overline{BC} \)

**To Prove**
\( mAD = ? \)
\( \text{Area of } \triangle ABC = ? \)
In a quadrilateral $ABCD$, the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other. 

Prove that:

$$m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2.$$ 

**Given:** Quadrilateral $ABCD$ diagonal $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other. 

**To Prove:** 

$$m(\overline{AB})^2 + m(\overline{CD})^2 = m(\overline{AD})^2 + m(\overline{BC})^2$$ 

**Proof**

<table>
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| In right triangle $AOB$ 
$m(\overline{AB})^2 = m(\overline{AO})^2 + m(\overline{OB})^2$ ....(i) | By Pythagoras theorem |
| In right triangle $COD$ 
$m(\overline{CD})^2 = m(\overline{OC})^2 + m(\overline{OD})^2$ ....(ii) | By Pythagoras theorem |
| In right triangle $AOD$ 
$m(\overline{AD})^2 = m(\overline{AO})^2 + m(\overline{OD})^2$ ....(iii) | By Pythagoras theorem |
| In right triangle $BOC$ 
$m(\overline{BC})^2 = m(\overline{OB})^2 + m(\overline{OC})^2$ ....(iv) | By Pythagoras theorem |
| $m(\overline{AB})^2 + m(\overline{CD})^2 = m(\overline{AO})^2 + m(\overline{OB})^2 + m(\overline{OC})^2 + m(\overline{OD})^2$ ....(v) | By adding (i) and (ii) |
| $m(\overline{AD})^2 + m(\overline{BC})^2 = m(\overline{AO})^2 + m(\overline{OD})^2 + m(\overline{OB})^2 + m(\overline{OC})^2$ ....(vi) | By adding (ii) and (iv) |
| $(m\overline{AB})^2 + (m\overline{CD})^2 = (m\overline{BC})^2 + (m\overline{AD})^2$ | By adding (v) and (vi) |

6. (i) **In the $\triangle ABC$ as shown in the figure**, $m\angle ACB = 90^\circ$ and $\overline{CD} \perp \overline{AB}$. Find the lengths $a$, $h$ and $b$ if $m\overline{BD} = 5$ units and $m\overline{AD} = 7$ units. 

**Given:** A $\triangle ABC$ as shown 
$m \angle ACB = 90^\circ$ 
and $\overline{CD} \perp \overline{AB}$ 

**To prove:** $a$, $h$ and $b$. 

In right angled $\triangle BDC$ 
$a^2 = 25 + h^2$ 

in right angled $\triangle ADC$ 
$b^2 = 49 + h^2$ 

in right angled $\triangle ABC$ 
$a^2 + b^2 = 144$ 

adding (i) and (ii) 

$$a^2 + b^2 = 74 + 2h^2$$
from (iii) and (iv)
\[ 74 + 2h^2 = 144 \]
\[ 2h^2 = 144 - 74 \]
\[ 2h^2 = 70 \]
\[ h^2 = 35 \]
\[ h = \sqrt{35} \]

Now we will find a and b
Put
\[ h^2 = 35 \text{ (in Eq. 1)} \]
\[ a^2 = 25 + 35 \]
\[ a^2 = 60 \]
\[ a = \sqrt{60} \]
\[ = \sqrt{4 \times 15} \]
\[ a = 2\sqrt{15} \]

now put
\[ h^2 = 35 \text{ (in Eq. 2)} \]
\[ b^2 = 49 + 35 \]
\[ b^2 = 48 \]
\[ b = \sqrt{48} \]
\[ = \sqrt{4 \times 21} \]
\[ b = 2\sqrt{21} \]

SO
\[ a = 2\sqrt{15} \]
\[ h = \sqrt{35} \]
\[ b = 2\sqrt{21} \]

(ii) Find the value of x in the shown in the figure.

In right angled triangle ADC
\[ m(AC)^2 = m(AD)^2 + m(DC)^2 \]
\[ (13)^2 = (AD)^2 + (5)^2 \]
\[ 169 = (AD)^2 + 25 \]
\[ (AD)^2 = 169 - 25 \]
\[ (AD)^2 = 144 \]
\[ AD = \sqrt{144} \]
\[ AD = 12 \text{ cm} \]

In right angled triangle ABD
\[ (AB)^2 = (AD)^2 + (BD)^2 \]
\[ (15)^2 = (12)^2 + x^2 \]
\[ 225 = 144 + x^2 \]
\[ x^2 = 225 - 144 \]
\[ x^2 = 81 \]
\[ x = 9 \text{ cm} \]

7. A plane is at a height of 300 m and is 500 m away from the airport as shown in the figure. How much distance will it travel to land at the airport?

Here A be the position of plane and B be the position of airport.

\[ mAC = 500 \text{ m} \]
\[ mBC = 300 \text{ m} \]
\[ mAB = ? \]

Applying Pythagoras theorem on right angled triangle ABC
\[ AB^2 = AC^2 + BC^2 \]
\[ = (500)^2 + (300)^2 \]
\[ = 250000 + 90000 \]
\[ = 340000 \]

so
\[ AB^2 = 34\times10000 \]
\[ AB = \sqrt{34\times10000} \]
\[ = \sqrt{34\times100\times100} \]
\[ = 100\sqrt{34} \text{m} \]

So required distance is \( 100\sqrt{34} \text{m} \)

8. A ladder 17 m long rests against a vertical wall. The foot of the ladder is 8 m away from the base of the wall. How high up the wall will the ladder reach?

**Ans.** Let the height of ladder = \( x \) m in right angled triangle

\[
\text{Hyp}^2 = (\text{Perp.})^2 + (\text{Base})^2
\]

\[
(17)^2 = (x)^2 + (8)^2
\]

\[
289 = x^2 + 64
\]

\[
x^2 = 289 - 64
\]

\[
x^2 = 225
\]

\[
x = \sqrt{225} = 15 \text{m}
\]

9. A student travels to his school by the route as shown in the figure. Find \( m\overline{AD} \), the direct distance from his house to school.

According to figure, \( m\overline{AB} = 2 \text{km} \)

\( m\overline{BC} = 6 \text{km} \)

\( m\overline{CD} = 3 \text{km} \)

Here \( m\overline{AB} \) and \( m\overline{CD} \) are perpendicular

Perpendicular = \( \overline{AB} + \overline{CD} \)

\[
= 2 + 3
\]

\[
= 5 \text{km}
\]

According to Pythagoras theorem

\[
(H)^2 = P^2 + B^2
\]

\[
(m\overline{AD})^2 = (5)^2 + (6)^2 = 25 + 36
\]

\[
(m\overline{AD})^2 = 61
\]

\[
m\overline{AD} = \sqrt{61} \text{Km}
\]

10. Which of the following are true and which are false?

(i) In a right angled triangle greater angle is 90°. (T)

(ii) In a right angled triangle right angle is 60°. (F)

(iii) In a right triangle hypotenuse is a side opposite to right angle. (T)

(iv) If \( a, b, c \) are sides of right angled triangle with \( c \) as longer side then \( c^2 = a^2 + b^2 \). (T)

(v) If 3 cm and 4 cm are two sides of a right angled triangle, then hypotenuse is 5 cm. (T)

(vi) If hypotenuse of an isosceles right triangle is \( \sqrt{2} \) cm then each of other side is of length 2 cm. (F)

11. Find the unknown value in each of the following figures.

(i)

\[
\begin{align*}
4 \text{ cm} \\
3 \text{ cm}
\end{align*}
\]

By Pythagoras theorem
(Hyp)^2 = (Perp.)^2 + (Base)^2
x^2 = (4)^2 + (3)^2
x^2 = 16 + 9
x^2 = 25 \Rightarrow x = \sqrt{25}
x = 5\text{cm}

By Pythagoras theorem
(Hyp)^2 = (Perp.)^2 + (Base)^2
(10)^2 = (6)^2 + (x)^2
100 = 36 + x^2
x^2 = 64
x = \sqrt{64}
X = 8\text{cm}

By Pythagoras theorem
(Hyp.)^2 = (Perp.)^2 + (Base)^2
(\sqrt{2})^2 = (x)^2 + (1)^2
x^2 = 2 - 1
x^2 = 1
x = \sqrt{1} = 1\text{cm}

OBJECTIVE

1. In a right angled triangle, the square of the length of hypotenuse is equal to the _____ of the squares of the lengths of the other two sides
   (a) Sum
   (b) Difference
   (c) Zero
   (d) None

2. If the square of one side of a triangle is equal to the sum of the squares of the other two sides then the triangle is a _____ triangle.
   (a) Right angled
   (b) Acute angled
   (c) Obtuse angled
   (d) None
3. Let c be the longest of the sides a, b and c of a triangle. If \( a^2 + b^2 = c^2 \), then the triangle is ____:
   (a) Right
   (b) Acute
   (c) Obtuse
   (d) None

4. Let c be the longest of the sides a, b and c of a triangle. If \( a^2 + b^2 > c^2 \), then triangle is:
   (a) Acute
   (b) Right
   (c) Obtuse
   (d) None

5. Let c be the longest of the sides a, b and c of a triangle of \( a^2 + b^2 < c^2 \), then the triangle is:
   (a) Acute
   (b) Right

6. If 3cm and 4cm are two sides of a right angled triangle, then hypotenuse is:
   (a) 5cm
   (b) 3cm
   (c) 4cm
   (d) 2cm

7. In right triangle ____ is a side opposite to right angle.
   (a) Base
   (b) Perpendicular
   (c) Hypotenuse
   (d) None

**ANSWER KEY**

1. a  
2. a  
3. (c)  
4. a  
5. c  
6. a  
7. c